AN ACCURATE LEGENDRE COLLOCATION SCHEME FOR COUPLED HYPERBOLIC EQUATIONS WITH VARIABLE COEFFICIENTS

E.H. DOHA[1,a], A.H. BHRAWY[2,3,b], D. BALEANU[4,5,6,d], M.A. ABDELKAWY[3,c]

[1]Department of Mathematics, Faculty of Science, Cairo University, Giza, Egypt,
E-mail[a]: eiddoha@frcu.eun.eg
[2]Department of Mathematics, Faculty of Science, King Abdulaziz University,
Jeddah 21589, Saudi Arabia,
E-mail[b]: alibhrawy@yahoo.co.uk
[3]Department of Mathematics, Faculty of Science, Beni-Suef University, Beni-Suef 62511, Egypt,
E-mail[c]: melkawy@yahoo.com
[4]Department of Chemical and Materials Engineering, Faculty of Engineering,
King Abdulaziz University, Jeddah 21589, Saudi Arabia
[5]Department of Mathematics and Computer Sciences, Cankaya University, Eskisehir Yolu 29.km,
06810 Ankara, Turkey
[6]Institute of Space Sciences, P.O.BOX, MG-23, RO-077125, Magurele-Bucharest, Romania,
E-mail[d]: dumitru@cankaya.edu.tr

Received December 23, 2013

The study of numerical solutions of nonlinear coupled hyperbolic partial differential equations (PDEs) with variable coefficients subject to initial-boundary conditions continues to be a major research area with widespread applications in modern physics and technology. One of the most important advantages of collocation method is the possibility of dealing with nonlinear partial differential equations (NPDEs) as well as PDEs with variable coefficients. A numerical solution based on a Legendre collocation method is extended to solve nonlinear coupled hyperbolic PDEs with variable coefficients. This approach, which is based on Legendre polynomials and Gauss-Lobatto quadrature integration, reduces the solving of nonlinear coupled hyperbolic PDEs with variable coefficients to a system of nonlinear ordinary differential equations that is far easier to solve. The obtained results show that the proposed numerical algorithm is efficient and very accurate.

Key words: Nonlinear coupled hyperbolic partial differential equations; Nonlinear phenomena; Collocation method; Gauss-Lobatto quadrature.

PACS: 02.30.Gp, 02.30.Hq, 02.30.Jr, 02.30.Mv, 02.60.-x.

1. INTRODUCTION

For several decades, analytical and numerical methods have been developed to obtain more accurate solutions of differential and integral equations [1]-[17]. The spectral method [18–22] is one of the family of weighted residual numerical methods for solving various problems, including variable coefficient and nonlinear differential equations [23, 24], integral equations [25, 26], fractional orders differential equations

Rom. Journ. Phys., Vol. 59, Nos. 5-6, P. 408–420, Bucharest, 2014

[27–29] and function approximation and variational problems [30]. The collocation method [31–34] can be classified as a special type of spectral methods. In the last few years, the collocation method has been introduced as a powerful approximation method for numerical solutions of all kinds of initial boundary-value problems.

Coupled hyperbolic systems have a wide ange of applications in physics [35–39], such as microwave processes, electromagnetism, electrodynamics, acoustics, thermo elasticity, electrical engineering, fluid dynamics, polymer science, reaction-diffusion, and population dynamics. Analytical study of variable coefficient mixed hyperbolic partial differential problems is discussed in [40]. Other numerical methods based on numerical integration techniques [41] are used to numerically solve different types of hyperbolic partial differential problems. In [42, 43], finite difference schemes are considered to numerically solve hyperbolic equations. Cubic B-spline collocation method on the uniform mesh points was used in [44] to numerically solve the nonlinear one-dimensional Klein-Gordon equation. There are also numerous results on studying the solitary and periodic wave solutions for several types of hyperbolic Klein-Gordon equations (see, for instance, Refs. [45–47]).

There are no results on Legendre-Gauss-Lobatto collocation (L-GL-C) method for solving nonlinear coupled hyperbolic PDEs with variable coefficients subject to initial boundary conditions. Therefore, the objective of this work is to present this method to numerically solve some nonlinear coupled hyperbolic PDEs with variable coefficients. By using collocation method, exponential convergence for the spatial variables can be achieved for approximating the solution of some PDEs. The computerized mathematical algorithm is the main key to apply this method for solving the problem. Two illustrative problems with various kinds of exact solutions such as triangular and soliton solutions are presented for demonstrating the high accuracy of this numerical scheme.

A brief outline of this paper is as follows. We present some properties of Legendre polynomials in the next section. In Section 3, we propose an efficient algorithm to solve coupled nonlinear hyperbolic PDEs with initial-boundary conditions. In Section 4, the proposed method is applied to two different test problems to show the accuracy of our method. In the last section, we present our conclusions.

2. LEGENDRE POLYNOMIALS

Some basic properties of Legendre polynomials have been recalled in this section. The Legendre polynomials $L_k(x)$ $(k=0,1\dots,)$ satisfy the following Rodrigues formula

$$L_k(x) = \frac{(-1)^k}{2^k k!} D^k((1-x^2)^k), \tag{1}$$

we recall also that $L_k(x)$ is a polynomial of degree k and therefore $L_k^{(q)}(x)$ (the q-th derivative of $L_k(x)$) is given by

$$L_k^{(q)}(x) = \sum_{i=0(k+i=even)}^{k-q} C_q(k,i) L_i(x), \tag{2}$$

where

$$C_q(k,i) = \frac{2^{q-1}(2i+1)\Gamma[\frac{q+k-i}{2}]\Gamma[\frac{q+k+i+1}{2}]}{\Gamma[q]\Gamma[\frac{2-q+k-i}{2}]\Gamma[\frac{3-q+k+i}{2}]}.$$

The Legendre polynomials satisfy the following relations

$$L_0(x) = 1, \quad L_1(x) = x, \quad L_{k+2}(x) = \frac{2k+3}{k+2} x L_{k+1}(x) - \frac{k+1}{k+2} L_k(x)$$

and the orthogonality relation

$$(L_k(x), L_l(x))_w = \int_{-1}^{1} L_k(x) L_l(x)\, w(x) = h_k \delta_{lk}. \tag{3}$$

where $w(x) = 1$, $h_k = \frac{2}{2k+1}$. The Legendre-Gauss-Lobatto quadrature has been used to evaluate the previous integrals accurately. For any $\phi \in S_{2N-1}[-1,1]$, we have that

$$\int_{-1}^{1} \phi(x) dx = \sum_{j=0}^{N} \varpi_{N,j} \phi(x_{N,j}). \tag{4}$$

We introduce the following discrete inner product

$$(u,v)_w = \sum_{j=0}^{N} u(x_{N,j})\, v(x_{N,j})\, \varpi_{N,j}, \tag{5}$$

where $x_{N,j}$ $(0 \le j \le N)$ and $\varpi_{N,j}$ $(0 \le j \le N)$ are used as the nodes and the corresponding Christoffel numbers in the interval $[-1,1]$, respectively.

3. THE PROBLEM AND THE NUMERICAL ALGORITHM

In this section, we approximate the solution of coupled nonlinear hyperbolic-type equations with variable coefficients for space variable by using the Legendre collocation method. In what follows, we propose an efficient numerical algorithm to solve the coupled nonlinear hyperbolic-type equations in the following form:

$$\begin{aligned} D_t^2 u(y,t) =&\gamma u(y,t)v(y,t)(D_y u(y,t)+D_t u(y,t)+D_y v(y,t)+D_t v(y,t)) \\ &+g_1(y,t)D_y^2 u(y,t)+g_2(y,t), \\ D_t^2 v(y,t) =&\delta u(y,t)v(y,t)(D_y u(y,t)+D_t u(y,t)+D_y v(y,t)+D_t v(y,t)) \\ &+g_3(y,t)D_y^2 v(y,t)+g_4(y,t), \quad (y,t)\in[A,B]\times[0,T], \end{aligned} \tag{6}$$

related to the initial conditions

$$\begin{aligned} &u(y,0)=f_1(y), \quad v(y,0)=f_2(y), \\ &D_t u(y,0)=f_3(y), \quad D_t v(y,0)=f_4(y), \quad y\in[A,B], \end{aligned} \tag{7}$$

and the boundary conditions

$$\begin{aligned} &u(A,t)=k_1(t), \quad u(B,t)=k_2(t), \\ &v(A,t)=k_3(t), \quad v(B,t)=k_4(t), \quad t\in[0,T]. \end{aligned} \tag{8}$$

We start with the transformations $x=\frac{2}{B-A}y+\frac{A+B}{A-B}, \quad w(x,t)=u(y,t), \quad z(x,t)=v(y,t)$. The problem (6)-(8) will be a new one in the spatial variable $x\in[-1,1]$. This transformation enables us to ease the using of the Legendre collocation method on $[-1,1]$,

$$\begin{aligned} D_t^2 w(x,t) =&\gamma w(x,t)z(x,t)\Big(\frac{2(D_x w(x,t)+D_x z(x,t))}{B-A}+D_t w(x,t)+D_t z(x,t)\Big) \\ &+\frac{4g_5(x,t)D_x^2 w(x,t)}{(B-A)^2}+g_6(x,t), \\ D_t^2 z(x,t) =&\delta w(x,t)z(x,t)\Big(\frac{2(D_x w(x,t)+D_x z(x,t))}{B-A}+D_t w(x,t)+D_t z(x,t)\Big) \\ &+\frac{4g_7(x,t)D_x^2 z(x,t)}{(B-A)^2}+g_8(x,t), \quad (x,t)\in[-1,1]\times[0,T], \end{aligned} \tag{9}$$

subject to a new set of initial and boundary conditions

$$\begin{aligned} &w(x,0)=f_5(x), \quad D_t w(x,0)=f_7(x), \\ &z(x,0)=f_6(x), \quad D_t z(x,0)=f_8(x), \quad x\in[-1,1], \end{aligned} \tag{10}$$

$$\begin{aligned} &w(-1,t)=k_1(t), \quad w(1,t)=k_2(t), \\ &z(-1,t)=k_3(t), \quad z(1,t)=k_4(t), \quad t\in[0,T]. \end{aligned} \tag{11}$$

Now, we are interested in using the L-GL-C method to transform the previous coupled PDEs into a system of ODEs. In order to do this, we approximate the spatial variable using L-GL-C method at some nodal points. The node points are the set of points in a specified domain where the dependent variable values are approximated. We take the roots of the Legendre orthogonal polynomials referred to as Legendre collocation points, which gives particularly accurate solutions for the spectral methods. Now, we outline the main steps of the L-GL-C method for solving a hyperbolic problem. Let us expand the dependent variable in a Legendre series,

$$w(x,t) = \sum_{j=0}^{N} a_j(t) L_j(x), \qquad z(x,t) = \sum_{j=0}^{N} b_j(t) L_j(x), \tag{12}$$

and in virtue of equations (3)-(5), we evaluate $a_j(t)$ and $b_j(t)$ by

$$a_j(t) = \frac{1}{h_j} \int_{-1}^{1} w(x,t)\, L_j(x) dx, \quad b_j(t) = \frac{1}{h_j} \int_{-1}^{1} z(x,t)\, L_j(x) dx. \tag{13}$$

For any positive integer N, $S_N[-1,1]$ stands for the set of polynomials of degree at most N. Thanks to (4), the coefficients $a_j(t)$ in terms of the solution at the collocation points can be approximated by

$$a_j(t) = \frac{1}{h_j} \sum_{i=0}^{N} L_j(x_{N,i}) \varpi_{N,i}\, w(x_{N,i},t), \quad b_j(t) = \frac{1}{h_j} \sum_{i=0}^{N} L_j(x_{N,i}) \varpi_{N,i}\, z(x_{N,i},t). \tag{14}$$

Due to (14), the approximate solution can be written as

$$\begin{aligned} w(x,t) &= \sum_{i=0}^{N} \Big(\sum_{j=0}^{N} \frac{1}{h_j} L_j(x_{N,i}) L_j(x) \varpi_{N,i} \Big) w(x_{N,i},t), \\ z(x,t) &= \sum_{i=0}^{N} \Big(\sum_{j=0}^{N} \frac{1}{h_j} L_j(x_{N,i}) L_j(x) \varpi_{N,i} \Big) z(x_{N,i},t). \end{aligned} \tag{15}$$

Furthermore, if we differentiate (15) once, and evaluate it at the first $N+1$ Legendre Gauss-Lobatto collocation points, it is easy to compute the first spatial partial derivative of the numerical solution in terms of the values at theses collocation points as

$$D_x w(x_{N,n},t) = \sum_{i=0}^{N} A_{ni}\, w(x_{N,i},t), \quad D_x z(x_{N,n},t) = \sum_{i=0}^{N} A_{ni}\, z(x_{N,i},t), \tag{16}$$

where

$$A_{ni} = \sum_{j=0}^{N} \frac{1}{h_j} L_j(x_{N,i}) \big(L_j(x)\big)' \varpi_{N,i}, \tag{17}$$

Proceeding as in the previous step, one can obtain the second spatial partial derivative as

$$D_x^2 w(x_{N,n},t) = \sum_{i=0}^{N} B_{ni} w(x_{N,i},t), \quad D_x^2 z(x_{N,n},t) = \sum_{i=0}^{N} B_{ni} z(x_{N,i},t), \tag{18}$$

where

$$B_{ni} = \sum_{j=0}^{N} \frac{1}{h_j} L_j(x_{N,i}) (L_j(x))'' \varpi_{N,i}. \tag{19}$$

In the proposed L-GL-C method the residual of (9) is set to zero at $N-1$ of Legendre-Gauss-Lobatto points, moreover, the boundary conditions (11) will be enforced at the two collocation points -1 and 1. Therefore, the approximation of (9)-(11) is

$$\begin{aligned}
\ddot{\mathrm{w}}_n(t) =& \frac{4 g_5(x_{N,n},t) g_6(x_{N,n},t) \sum\limits_{i=0}^{N} B_{ni} w_i(t)}{(B-A)^2} - \gamma w_n(t) z_n(t) (\dot{\mathrm{w}}_n(t) + \dot{\mathrm{z}}_n(t)) \\
&+ \frac{2\gamma w_n(t) z_n(t) \Big(\sum\limits_{i=0}^{N} A_{ni} (w_i(t) + z_i(t)) \Big)}{B-A} \\
\ddot{\mathrm{z}}_n(t) =& \frac{4 g_7(x_{N,n},t) g_8(x_{N,n},t) \sum\limits_{i=0}^{N} B_{ni} z_i(t)}{(B-A)^2} - \delta w_n(t) z_n(t) (\dot{\mathrm{w}}_n(t) + \dot{\mathrm{z}}_n(t)) \\
&+ \frac{2\delta w_n(t) z_n(t) \Big(\sum\limits_{i=0}^{N} A_{ni} (w_i(t) + z_i(t)) \Big)}{B-A}, \quad n = 1, \cdots, N-1,
\end{aligned} \tag{20}$$

where $w_k(t) = w(x_{N,k},t), \quad z_k(t) = z(x_{N,k},t), \quad k = 1, \cdots, N-1$. This approach provides a $(2N-2)$ system of second order ODEs in the expansion coefficients $a_j(t)$,

$b_j(t)$,

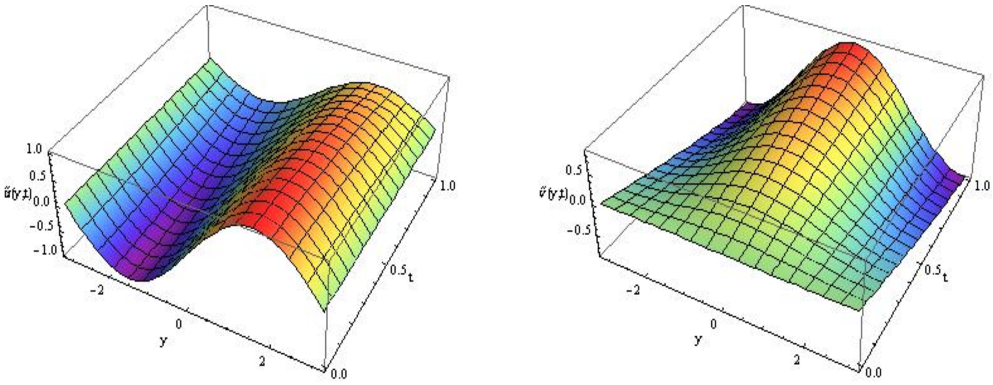
$$\begin{aligned}\ddot{w}_n(t) =& \frac{4\,g_5(x_{N,n},t)g_6(x_{N,n},t)\sum\limits_{i=0}^{N} B_{ni}w_i(t)}{(B-A)^2} - \gamma w_n(t)z_n(t)(\dot{w}_n(t)+\dot{z}_n(t))\\ &+ \frac{2\gamma\, w_n(t)z_n(t)\Big(\sum\limits_{i=0}^{N} A_{ni}(w_i(t)+z_i(t))\Big)}{B-A}\\ \ddot{z}_n(t) =& \frac{4\,g_7(x_{N,n},t)g_8(x_{N,n},t)\sum\limits_{i=0}^{N} B_{ni}z_i(t)}{(B-A)^2} - \delta w_n(t)z_n(t)(\dot{w}_n(t)+\dot{z}_n(t))\\ &+ \frac{2\delta w_n(t)z_n(t)\Big(\sum\limits_{i=0}^{N} A_{ni}(w_i(t)+z_i(t))\Big)}{B-A},\end{aligned} \tag{21}$$

with the following initial conditions $w_n(0) = f_5(x_{N,n})$, $\dot{w}_n(0) = f_7(x_{N,n})$, $z_n(0) = f_6(x_{N,n})$, and $\dot{z}_n(0) = f_8(x_{N,n})$. Otherwise, we can write the previous system as:

$$\begin{pmatrix} \ddot{w}_1(t)\\ \ddot{w}_2(t)\\ \cdots\\ \cdots\\ \ddot{w}_{N-1}(t)\\ \ddot{z}_1(t)\\ \ddot{z}_2(t)\\ \cdots\\ \cdots\\ \ddot{z}_{N-1}(t)\end{pmatrix} = \begin{pmatrix} F_1(t,w(t),z(t),\dot{w}(t),\dot{z}(t))\\ F_2(t,w(t),z(t),\dot{w}(t),\dot{z}(t))\\ \cdots\\ \cdots\\ F_{N-1}(t,w(t),z(t))\\ G_1(t,w(t),z(t),\dot{w}(t),\dot{z}(t))\\ G_2(t,w(t),z(t),\dot{w}(t),\dot{z}(t))\\ \cdots\\ \cdots\\ G_{N-1}(t,w(t),z(t))\end{pmatrix} \tag{22}$$

with

$$\begin{pmatrix} w_1(0)\\ w_2(0)\\ \cdots\\ \cdots\\ w_{N-1}(0)\\ z_1(0)\\ z_2(0)\\ \cdots\\ \cdots\\ z_{N-1}(0)\end{pmatrix} = \begin{pmatrix} f_5(x_{N,1})\\ f_5(x_{N,2})\\ \cdots\\ \cdots\\ f_5(x_{N,N-1})\\ f_6(x_{N,1})\\ f_6(x_{N,2})\\ \cdots\\ \cdots\\ f_6(x_{N,N-1})\end{pmatrix};\ \begin{pmatrix} \dot{w}_1(0)\\ \dot{w}_2(0)\\ \cdots\\ \cdots\\ \dot{w}_{N-1}(0)\\ \dot{z}_1(0)\\ \dot{z}_2(0)\\ \cdots\\ \cdots\\ \dot{z}_{N-1}(0)\end{pmatrix} = \begin{pmatrix} f_7(x_{N,1})\\ f_7(x_{N,2})\\ \cdots\\ \cdots\\ f_7(x_{N,N-1})\\ f_8(x_{N,1})\\ f_8(x_{N,2})\\ \cdots\\ \cdots\\ f_8(x_{N,N-1})\end{pmatrix}, \tag{23}$$



(a) (b)

Fig. 1 – (a) The space-time graph of the approximate solution $\widetilde{u}$ of problem (25) at $N = 12$; (b) The space-time graph of the approximate solution $\widetilde{v}$ of problem (25) at $N = 12$.

where

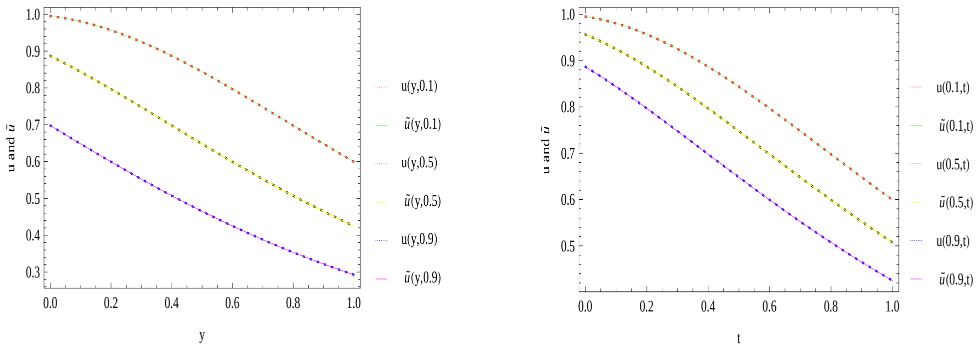
$$F_n(t,w(t),z(t)) = \frac{4\,g_5(x_{N,n},t)g_6(x_{N,n},t)\sum_{i=0}^{N} B_{ni}w_i(t)}{(B-A)^2} - \gamma w_n(t)z_n(t)(\dot{w}_n(t)+\dot{z}_n(t)) + \frac{2\gamma\, w_n(t)z_n(t)\Big(\sum_{i=0}^{N} A_{ni}(w_i(t)+z_i(t))\Big)}{B-A},$$

$$G_n(t,w(t),z(t)) = \frac{4\,g_7(x_{N,n},t)g_8(x_{N,n},t)\sum_{i=0}^{N} B_{ni}z_i(t)}{(B-A)^2} - \delta w_n(t)z_n(t)(\dot{w}_n(t)+\dot{z}_n(t)) + \frac{2\delta w_n(t)z_n(t)\Big(\sum_{i=0}^{N} A_{ni}(w_i(t)+z_i(t))\Big)}{B-A}. \tag{24}$$

The system of second order (22)-(23) can be solved by using diagonally-implicit Runge-Kutta-Nyström (DIRKN) method.

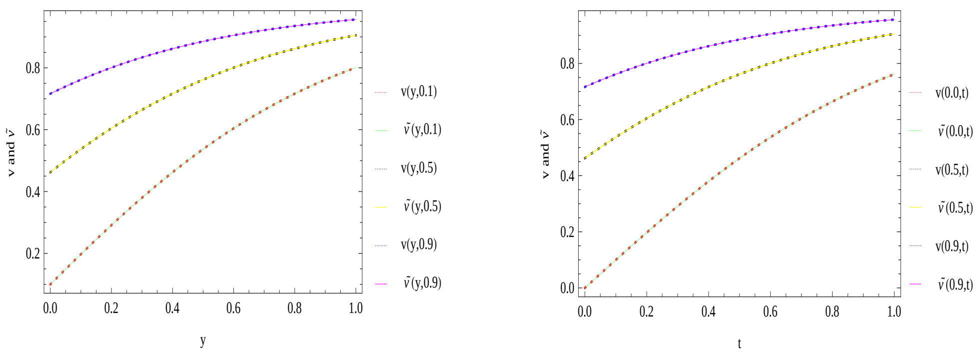
4. TEST PROBLEMS

We test the numerical accuracy of the proposed method by introducing two test problems with different types of exact solutions.



(a) (b)

Fig. 2 – (a) The curves of approximate $\widetilde{u}$ and the exact u solutions for different values of t=0.1, 0.5, and 0.9 of problem (32) where $N = 12$ in the interval [0,1]; (b) The curves of approximate $\widetilde{u}$ and the exact u solutions for different values of y=0.1, 0.5, and 0.9 of problem (32) where $N = 12$ in the interval [0,1].



(a) (b)

Fig. 3 – (a) The curves of approximate $\widetilde{v}$ and the exact v solutions for different values of t=0.1, 0.5, and 0.9 of problem (32) where $N = 12$ in the interval [0,1]; (b) The curves of approximate $\widetilde{v}$ and the exact v solutions for different values of y=0.0, 0.5, and 0.9 of problem (32) where $N = 12$ in the interval [0,1].

Table 1

N	MAE_1	$RMSE_1$	Ne_1	MAE_2	$RMSE_2$	Ne_2
4	3.73×10^{-4}	2.88×10^{-4}	7.93×10^{-4}	3.44×10^{-4}	1.47×10^{-4}	4.04×10^{-4}
8	2.02×10^{-8}	6.37×10^{-9}	1.74×10^{-8}	7.68×10^{-8}	2.22×10^{-8}	6.05×10^{-8}
12	1.52×10^{-8}	2.87×10^{-9}	7.81×10^{-9}	1.86×10^{-8}	1.19×10^{-8}	3.24×10^{-8}

4.1. TRIANGULAR SOLUTION

As a first example, we consider the coupled nonlinear hyperbolic equations (6) with the following functions

$$\begin{aligned} g_1(y,t) &= \left(1+e^t\cos(y)\right), \quad g_2 = \frac{1}{2}\cos(t)\left(e^t - 2\gamma\cos(t+y)\sin(t)\right)\sin(2y), \\ g_3(y,t) &= \left(1+e^t\sin(y)\right), \quad g_4 = \frac{1}{2}\left(e^t - 2\delta\cos(t)\cos(t+y)\right)\sin(t)\sin(2y), \end{aligned} \tag{25}$$

subject to

$$\begin{aligned} &k_1(t) = \sin(A)\cos(t), \quad k_2(t) = \sin(B)\cos(t), \\ &k_3(t) = \sin(t)\cos(A), \quad k_4(t) = \cos(B)\sin(t), \\ &f_1(t) = \sin(y), \quad f_2(t) = f_3(t) = 0, \quad f_4(t) = \cos(y). \end{aligned} \tag{26}$$

The exact solutions of this problem are

$$u(y,t) = \sin(y)\cos(t), \quad v(y,t) = \sin(t)\cos(y). \tag{27}$$

The absolute errors in the given tables are

$$E(y,t) = |u(y,t) - \widetilde{u}(y,t)|, \tag{28}$$

where $u(y,t)$ and $\widetilde{u}(y,t)$ are the exact and approximate solutions at the point (y,t), respectively. Moreover, the maximum absolute error is given by

$$M_E = \text{Max}\{E(y,t) : \forall (y,t) \in [A,B]\times[0,T]\}. \tag{29}$$

The root-mean-square (RMS) and N_e errors may be given by:

$$\text{RMS} = \sqrt{\sum_{i=0}^{N} \frac{(u(x_{N,i}),t_i) - \widetilde{u}(x_{N,i}),t_i)^2}{N+1}}, \tag{30}$$

$$N_e = \sqrt{\frac{\sum_{i=0}^{N}(u(x_{N,i}),t_i) - \widetilde{u}(x_{N,i}),t_i)^2}{\sum_{i=0}^{N}(u(x_{N,i}),t_i)}}. \tag{31}$$

Table 2

N	MAE_1	$RMSE_1$	Ne_1	MAE_2	$RMSE_2$	Ne_2
4	1.35×10^{-3}	4.35×10^{-4}	6.13×10^{-4}	2.63×10^{-3}	1.51×10^{-3}	2.15×10^{-3}
8	1.61×10^{-6}	9.88×10^{-7}	1.40×10^{-6}	9.83×10^{-7}	4.12×10^{-7}	5.84×10^{-7}
12	7.44×10^{-8}	3.33×10^{-8}	4.70×10^{-8}	9.30×10^{-8}	3.95×10^{-8}	5.59×10^{-8}

Maximum absolute, root-mean-square and N_e errors of (25) are introduced in Table1 using L-GL-C method with three different choices of N, in the interval [0,1]. The approximate solutions $\widetilde{u}$ and $\widetilde{v}$ of problem (25) at $N = 12$ have been plotted in Figs. 1(a) and 1(b), respectively.

4.2. SOLITON SOLUTION

Second, we consider the coupled nonlinear hyperbolic equation (6) with the following functions

$$\begin{aligned}
&g_1(y,t) = \left(1+e^t\cos(y)\right), \quad g_3(y,t) = \left(1+e^t\sin(y)\right),\\
&g_4 = -2\,\mathrm{sech}(y+t)^2\left(\mathrm{sech}(y+t)-e^t\sin(y)-\tanh(y+t)\right)\tanh(y+t),\\
&g_2 = \mathrm{sech}(y+t)(\mathrm{sech}(y+t)-\tanh(y+t))\times\\
&\qquad\left(-2\,\mathrm{sech}(y+t)\tanh(y+t)+e^t\cos(y)(\mathrm{sech}(y+t)+\tanh(y+t))\right),
\end{aligned} \tag{32}$$

subject to

$$\begin{aligned}
&k_1(t) = \mathrm{sech}(A+t), \quad k_2(t) = \mathrm{sech}(B+t),\\
&k_3(t) = \tanh(A+t), \quad k_4(t) = \tanh(B+t),\\
&f_1(t) = \mathrm{sech}(y), \quad f_2(t) = \tanh(y),\\
&f_3(t) = -\mathrm{sech}(y)\tanh(y), \quad f_4(t) = \mathrm{sech}(y)^2.
\end{aligned} \tag{33}$$

The exact solutions are

$$u(y,t) = \mathrm{sech}(y+t), \quad v(y,t) = \tanh(y+t). \tag{34}$$

In Table2, we show the maximum absolute, root-mean-square and N_e errors of (32) for various choices of N, in the interval [0,1]. We plotted the curves of approximate and exact solutions of $\widetilde{u}$ at different values of y and t in Figs. 2(a) and 2(b), respectively. Moreover, from Figs. 3(a) and 3(b), we see that the curves of the approximate and exact solutions ($\widetilde{v}$ and v) almost coincide for different values of y and t that are listed in their captions.

5. CONCLUSION

We have proposed an efficient and accurate numerical algorithm based on Legendre-Gauss-Lobatto spectral method to get high accurate solutions for nonlinear coupled hyperbolic equations. The method is based upon reducing the mentioned problem into a system of second order ODEs in the expansion coefficient of the solution. The use of the Legendre-Gauss-Lobatto points as collocation nodes saves the spectral convergence for the spatial variable in the approximate solution. Numerical examples were also provided to illustrate the effectiveness of the derived numerical algorithm.

REFERENCES

1. D. Kumar, J. Singh, and Sushila, Rom. Rep. Phys. **65**, 63 (2013).
2. D. Rostamy, M. Alipour, H. Jafari, and D. Baleanu, Rom. Rep. Phys. **65**, 334 (2013).
3. D. Baleanu, A.H. Bhrawy, and T.M. Taha, Abstr. Appl. Anal. **2013**, Article ID 546502 (2013).
4. F.A. Aliev, N.I. Velieva, Y.S. Gasimov, N.A. Safarova, and L.F. Agamalieva, Proc. Romanian Acad. A **13**, 207 (2012).
5. J.A. Tenreiro Machado, P. Stefanescu, O. Tintareanu, and D. Baleanu, Rom. Rep. Phys. **65**, 316 (2013).
6. H. Leblond, H. Triki, and D. Mihalache, Rom. Rep. Phys. **65**, 925 (2013).
7. G. Ebadi *et al.*, Rom. J. Phys. **58**, 3 (2013).
8. A.M. Wazwaz, Rom. J. Phys. **58**, 685 (2013).
9. H. Leblond and D. Mihalache, Phys. Reports **523**, 61 (2013).
10. L. Girgis, D. Milovic, S. Konar, A. Yildirim, H. Jafari, and A. Biswas, Rom. Rep. Phys. **64**, 663 (2012).
11. Y.J. He and D. Mihalache, Rom. Rep. Phys. **64**, 1243 (2012).
12. D. Mihalache, Rom. J. Phys. **57**, 352 (2012).
13. A. Biswas, A. Yildirim, T. Hayat, O.M. Aldossary, and R. Sassaman, Proc. Romanian Acad. A **13**, 32 (2012).
14. G. Ebadi, A.H. Kara, M.D. Petkovic, A. Yildirim, and A. Biswas, Proc. Romanian Acad. A **13**, 215 (2012).
15. B. Ahmed and A. Biswas, Proc. Romanian Acad. A **14**, 111 (2013).
16. Guangye Yang *et al.*, Rom. Rep. Phys. **65**, 902 (2013).
17. Guangye Yang *et al.*, Rom. Rep. Phys. **65**, 391 (2013).
18. K. Krastev and M. Schäfer, Comptes Rendus Mécanique **333**, 59 (2005).
19. M.R. Schumack, W.W. Schultz, and J.P. Boyd, J. Comput. Phys. **94**, 30 (1991).
20. E.H. Doha and A.H. Bhrawy, Comput. Math. Appl. **64**, 558 (2012).
21. E. Tohidi, A.H. Bhrawy, and K. Erfani, Appl. Math. Modell. **37**, 4283 (2013).
22. K. Schneider, S. Neffaa, and W.J.T. Bos, Comput. Phys. Commun. **182**, 2 (2011).
23. A.H. Bhrawy, Appl. Math. Comput. **222**, 255 (2013).
24. Alaeddin Malek, Nonlinear Analysis: Theory, Methods & Applications **30**, 4805 (1997)
25. A.H. Bhrawy, E. Tohidi, and F. Soleymani, Appl. Math. Comput. **218**, 10848 (2012).
26. K. Maleknejad, B. Basirat, and E. Hashemizadeh, Math. Comput. Modell. **55**, 1363 (2012).

27. A.H. Bhrawy, M.M. Tharwat, and A. Yildirim, Appl. Math. Model. **37**, 4245 (2013).
28. E.H. Doha, A.H. Bhrawy, D. Baleanu, and S.S. Ezz-Eldien, Appl. Math. Comput. **219**, 8042 (2013).
29. A.H. Bhrawy and A.S. Alofi, Appl. Math. Lett. **26**, 25 (2013).
30. C. Canuto, M.Y. Hussaini, A. Quarteroni, and T.A. Zang, *Spectral Methods: Fundamentals in Single Domains* (Springer-Verlag, New York, 2006).
31. E.H. Doha, A.H. Bhrawy, R.M. Hafez, and M.A. Abdelkawy, Appl. Math. Inf. Sci. **8**, 535 (2014)
32. E.H. Doha, D. Baleanu, A.H. Bhrawy, and M.A. Abdelkawy, Abstract and Applied Analysis **2013**, 760542 (2013).
33. Ya-Song Sun and Ben-Wen Li, J. Quant. Spectrosc. Radiat. Transfer **113**, 2205 (2012).
34. Minoo Kamrani and S. Mohammad Hosseini, Math. Comput. Simulat. **82**, 1630 (2012).
35. A.V. Nedzwedzky and O.L. Samborsky, Phys. Lett. A **210**, 85 (1996).
36. W.D. Bastos, A. Spezamiglio, and C.A. Raposo, J. Math. Anal. Appl. **381**, 557 (2011).
37. A. F. Harvey, *Microwave Engineering* (Academic Press, New York, 1963).
38. K.R. Khusnutdinova and D.E. Pelinovsky, Wave Motion **38**, 1 (2003).
39. O.M. Braun and Yu.S. Kivshar, Phys. Reports **306**, 1 (1998).
40. M.J. Rodriguez-Alvarez, G. Rubio, and L. Jodar, Appl. Math. Lett. **16**, 309 (2003).
41. E. Ponsoda, L. Jódar, and S. Jerez, Comput. Math. Appl. **47**, 233 (2004).
42. M.A.M. Lynch, Appl. Numer. Math. **31**, 173 (1999).
43. X. Cui, J. Yue, and Guang-wei Yuan, J. Comput. Appl. Math. **235**, 3527 (2011).
44. J. Rashidinia, M. Ghasemi, and R. Jalilian, J. Comput. Appl. Math. **233**, 1866 (2010).
45. Z.-Y. Zhang, J. Zhong, S.S. Dou, J. Liu, D. Peng, and T. Gao, Rom. J. Phys. **58**, 749 (2013).
46. Z.-Y. Zhang, J. Zhong, S.S. Dou, J. Liu, D. Peng, and T. Gao, Rom. J. Phys. **58**, 766 (2013).
47. G. Murariu and A. Ene, Rom. J. Phys. **53**, 651 (2008).